

Ph.D. Thesis Defense

On Large-Scale Multiparty Computation with sub-linear
Communication using Ephemeral Servers

Anders Konring, IT University of Copenhagen

November 19, 2023

Encryption to the Future [CDK+22]

A Paradigm for Sending Secret Messages to Future (Anonymous) Committees

Matteo Campanelli, Bernardo David, Hamidreza Khoshakhlagh, Anders Konring, Jesper Buus Nielsen

YOLO YOSO [CDGK22]

Fast and Simple Encryption and Secret Sharing in the YOSO Model

Ignacio Cascudo, Bernardo David, Lydia Garms, Anders Konring

Layered MPC [DDG+23]

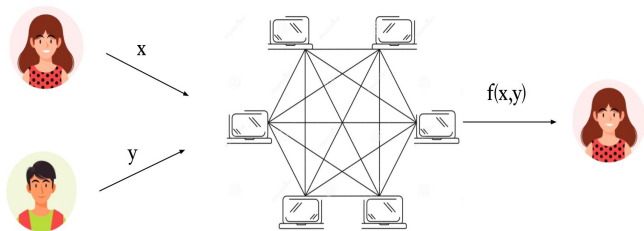
Perfect MPC over Layered Graphs

Bernardo David, Yuval Ishai, Anders Konring, Eyal Kushilevitz, Varun Narayanan
(Aarushi Goel, Chen-Da Liu-Zhang, Giovanni Deligios)

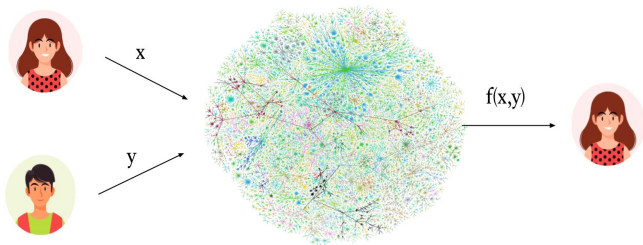
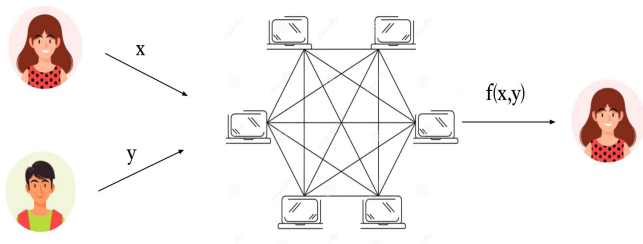
IT UNIVERSITY OF COPENHAGEN



Multiparty Computation (MPC)

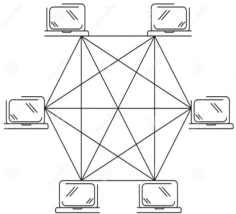


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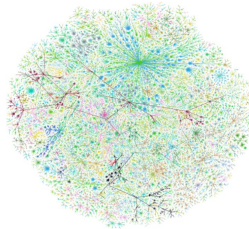


MPC Environments

MPC (classic environment):



MPC (dynamic environment):

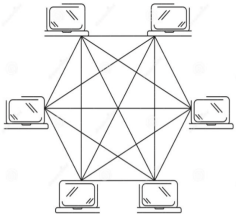


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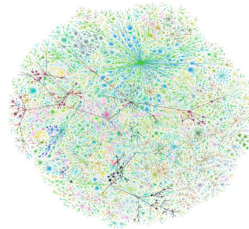
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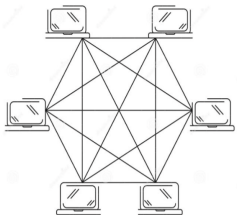


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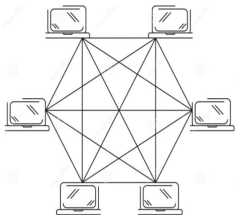


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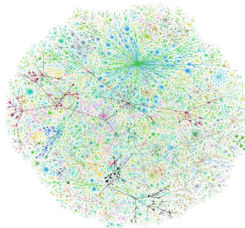
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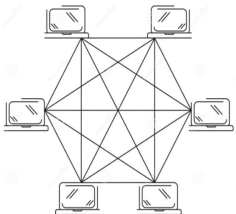


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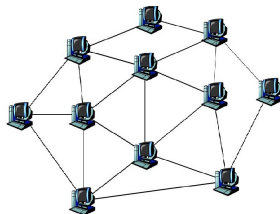
1. Large-Scale MPC on Blockchains
2. YOSO MPC [GHK+21]
 - Mobile Adversary
 - Role Assignment (RA) and Role Execution (RX)
3. Overview
4. Contributions
 - Encryption to the Future [CDK+22]
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Large-Scale MPC on Blockchains

Permissionless Blockchains

Blockchains are large public P2P networks.

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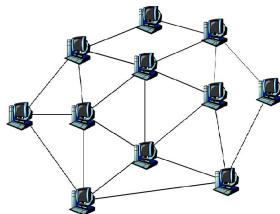


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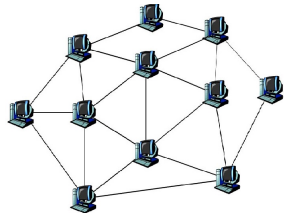
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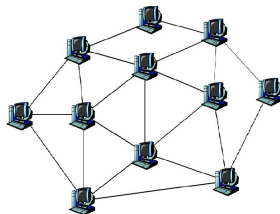
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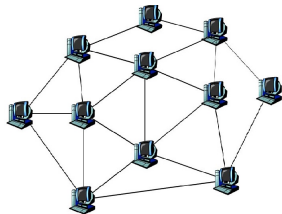
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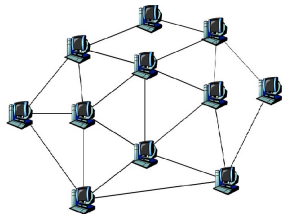
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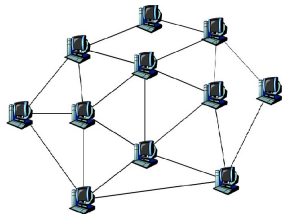
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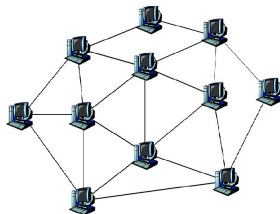
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YES! [BGG+20, GHK+21, CGG+21]

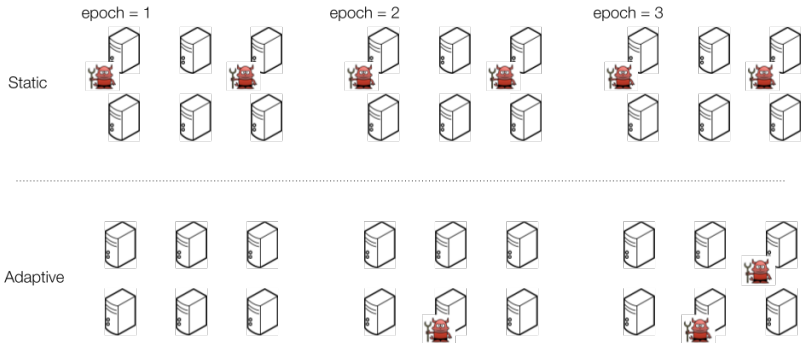


YOSO MPC [GHK⁺21]

Mobile Adversary



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Mobile Adversary [OY91]

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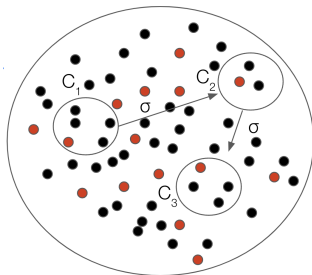
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- Decades of research in Proactive Secret Sharing and MPC.
- But existing work either settles for
 - multi-round epochs, $|\text{epochs}| > 1$ [HJKY95, ADN06, BELO15, ELL20]
 - corruption threshold n/c [OY91]
 - generally incompatible with large networks (thousands/millions) of nodes.



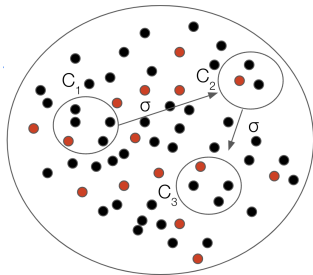
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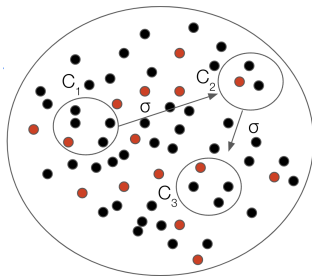


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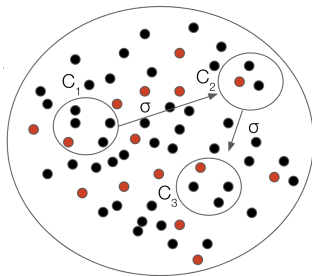


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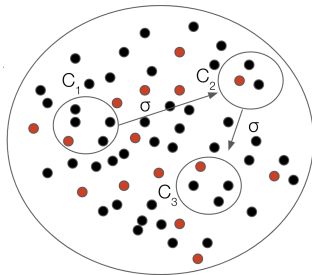


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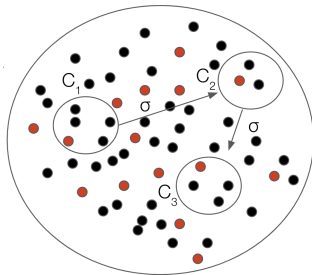
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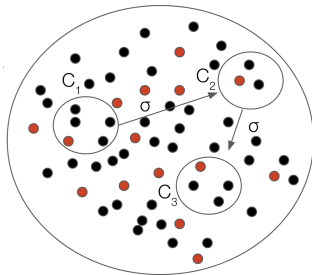
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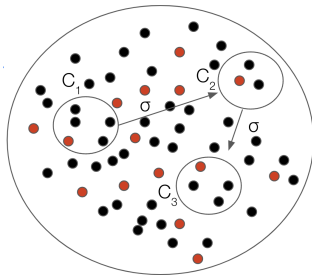
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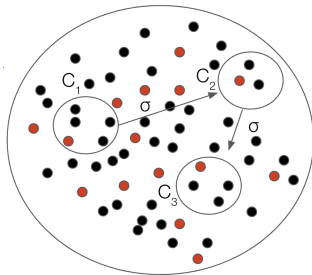
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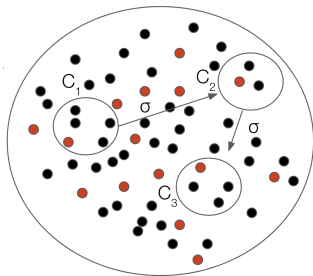
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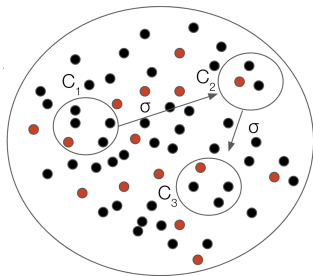
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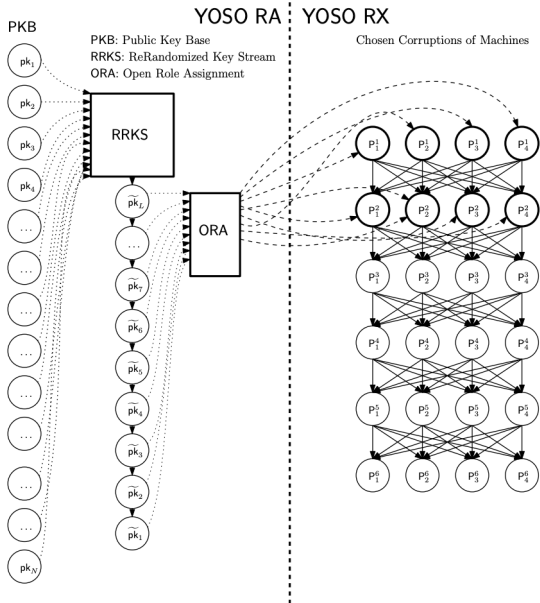
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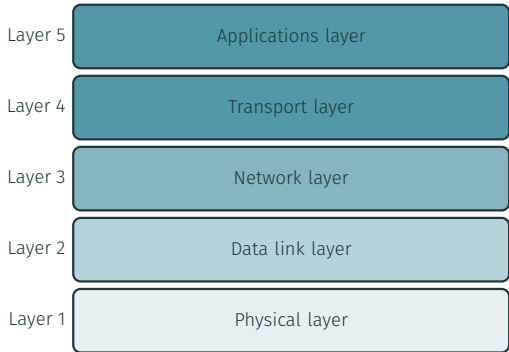
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Overview



OSI Model



Ephemeral Committees Model

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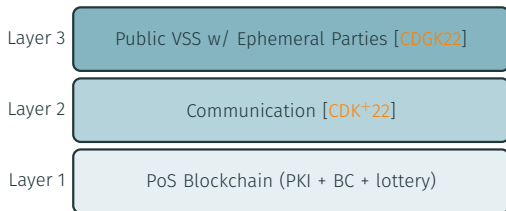
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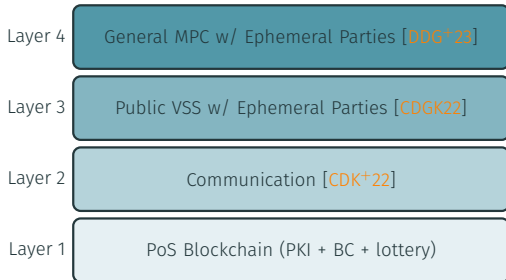
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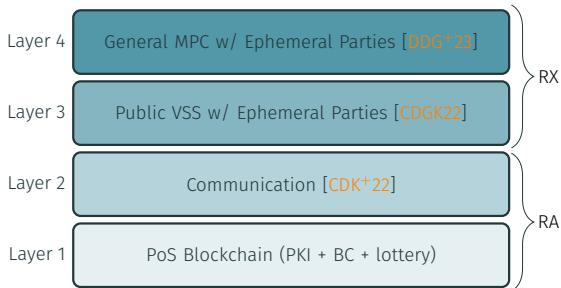
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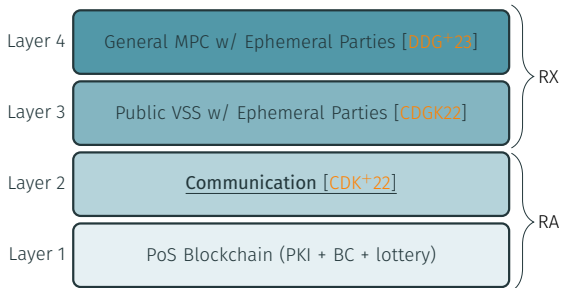
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- General Witness Encryption [GGSW13]
Sufficient for implementing any RA mechanism but constructions rely on existence of efficient multi-linear maps or iO

Overview: Encryption to the Future [CDK+22]

Existing work on (RA):

- YOSO RA (toy example) [GHK+21]
A blueprint for implementing RA but too concrete to generalize the problem of transferring secret state to future committees.
- Random-Index PIR [GHM+21]
Implementation of RA using the YOSO RA blueprint but relies on strong assumptions such as Mixnets or FHE.
- General Witness Encryption [GGSW13]
Sufficient for implementing any RA mechanism but constructions rely on existence of efficient multi-linear maps or iO

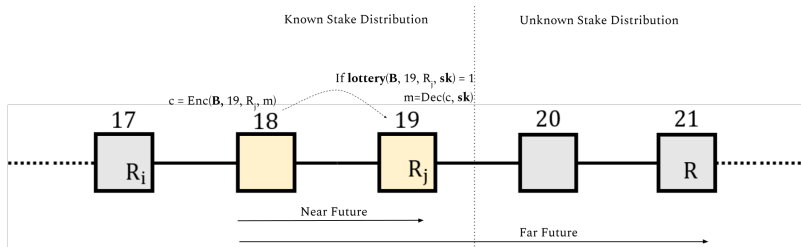
Motivation: Transferring secret state to future committees

- Consider secret state to the "near" vs. "far" future.
- Investigate the need for auxiliary committees for carrying state into the future.
- Consider the need for authenticated channels (Authentication-from-the-Past).
- Possibility of realizing RA using "standard" assumptions.

Overview: Encryption to the Future [CDK+22]

Encryption to the near Future.

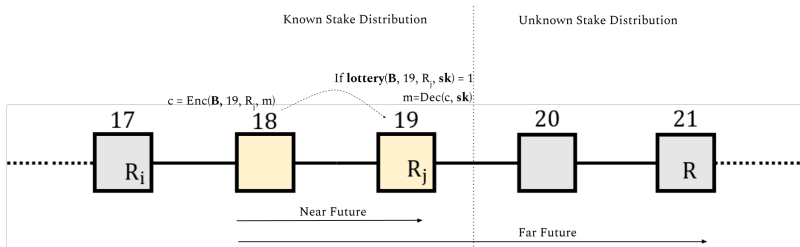
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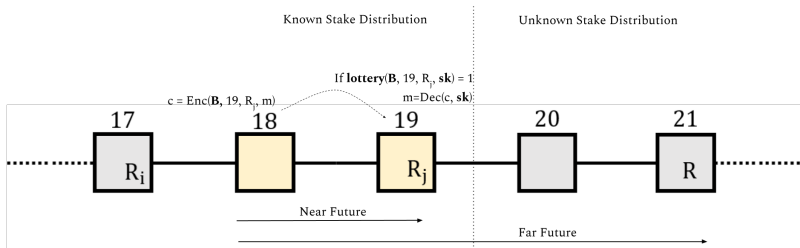
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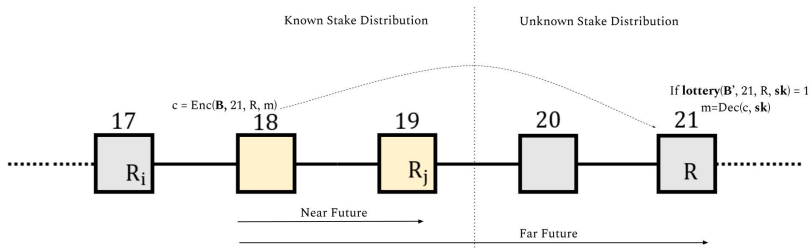
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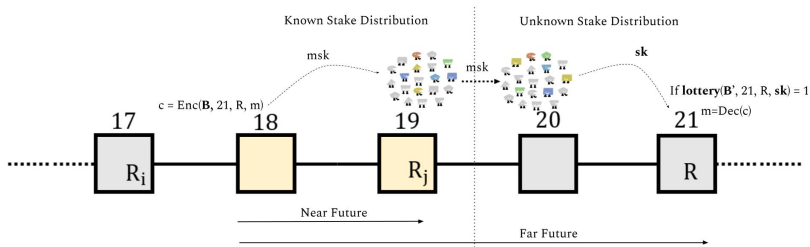
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Encryption to the far Future.

1. No auxiliary committees \iff BWE (Blockchain Witness Encryption).
2. Construction using EtF (near) + TIBE. With minimal use of auxiliary committees (indep. of size/number of messages)



Overview: Encryption to the Future [CDK+22]

Type	Scheme	Communication	Committee?	Interaction?
EtF (near)	CaBKaS [BGG+20]	$O(1)$	yes	yes
	RPIR [GHK+21]	$O(1)$	yes	yes
	cWE(GC+OT) (Sec. 4.2)	$O(N)$	no	no*
EtF (far)	IBE (Sec. 7)	$O(1)$	yes	yes
	WEB [GKM+20]	$O(M)$	yes	yes
	Full-fledged WE	$O(1)$	no	no

- “Committee?” indicates whether a committee is required.
- “Communication” refers to the communication complexity in the number of all parties N , or the number of plaintexts (called deposited secrets in [GKM+20]) M of a given fixed length.
- Asterisk* means non-interactive solutions that require sending a first reusable message

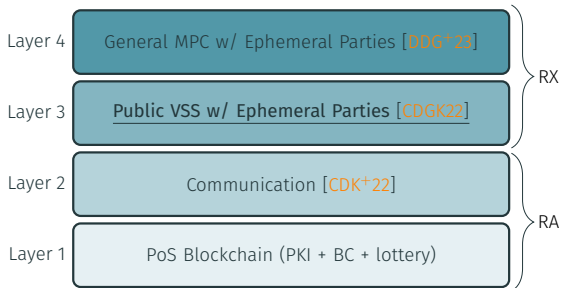
Overview

Layer 4 (Layered MPC):
Perfect General MPC over a layered graph using only ephemeral servers.

Layer 3 (YOLO-YOSO):
PVSS and resharing - basic building block for MPC and other applications.

Layer 2 (Encryption to the Future):
Communication towards unknown lottery winners.

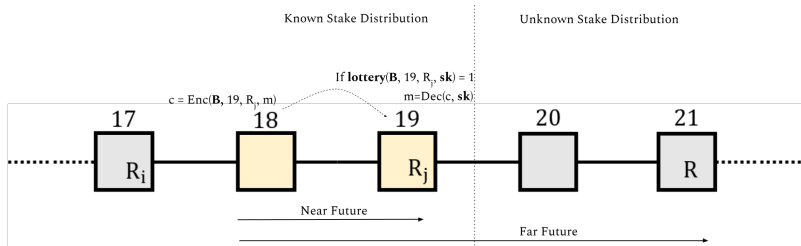
Layer 1:
Public PoS blockchain such as Ouroboros Praos [DGKR18].



Ephemeral Committees Model

Encryption to the near Future (Revisited).

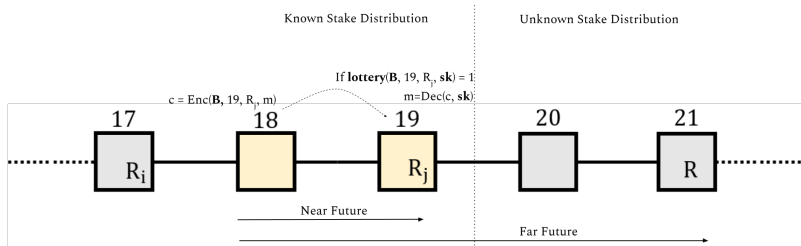
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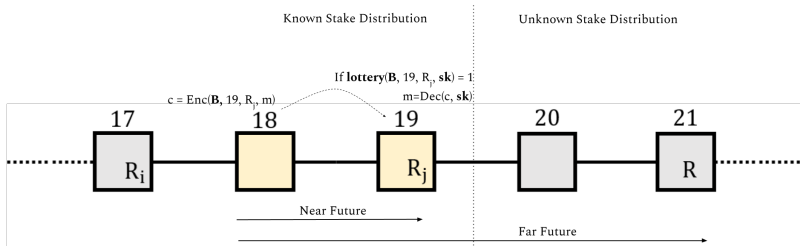
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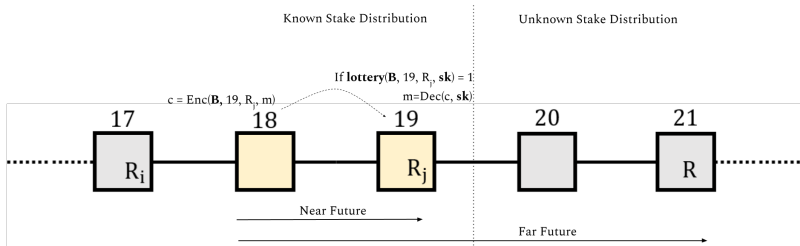
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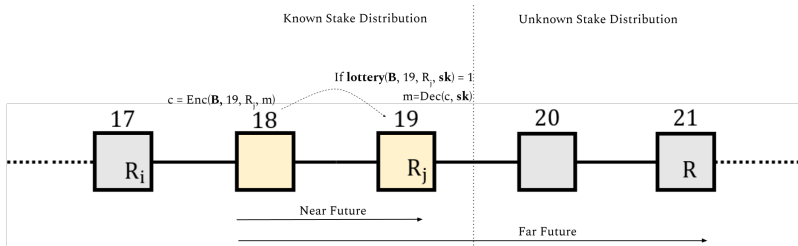
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Main Question

Can we design an EtF (near) scheme with $O(1)$ ciphertext length and allows for building practical PVSS (amenable to efficient techniques for proof of correct sharing)?



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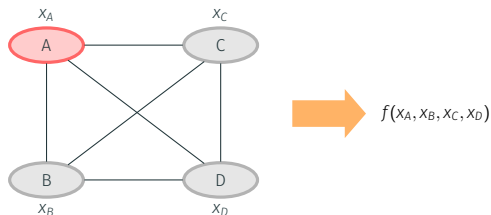
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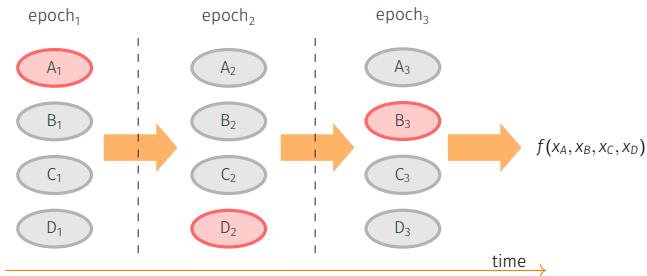
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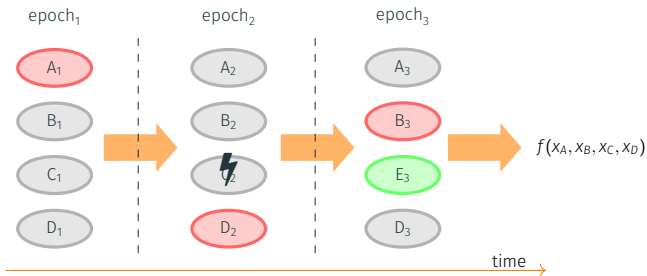
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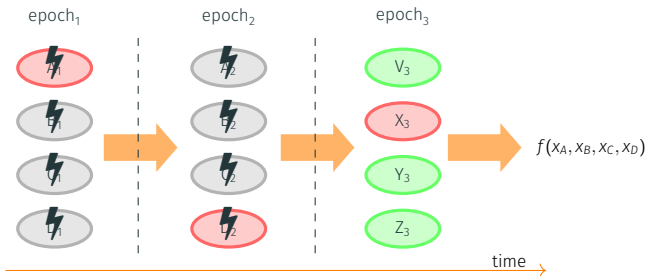
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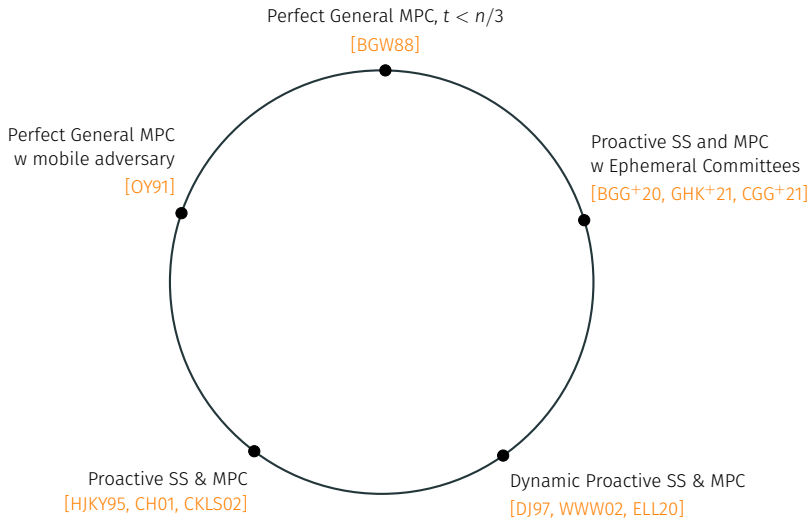


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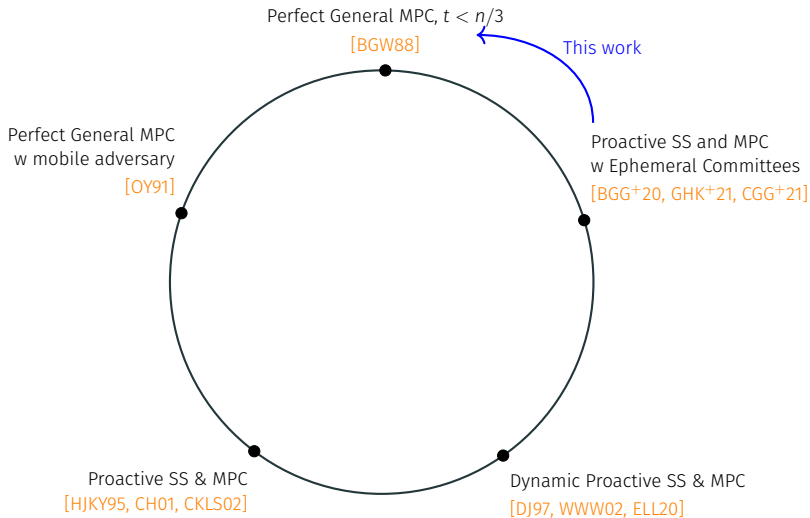
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Overview: Layered MPC [DDG⁺23]



Overview: Layered MPC [DDG⁺23]



*Is it possible to construct MPC with ephemeral committees achieving **perfect full security** against a maximally mobile adversary* while maintaining **optimal corruption threshold**?*

Overview: Layered MPC [DDG⁺23]

Area	Reference	epoch	Security	Corruption	Setup (BC+Chan.)
Proactive MPC	[HJKY95]	>1	Comp (full)	$t < n/2$	Next Round
	[OY91]	=1	Stat (full)	$t < n/c^t$	Next Round
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	<u>This work</u>	<u>=1</u>	<u>Perfect (full)</u>	<u>$t < n/3$</u>	<u>Next Round</u>

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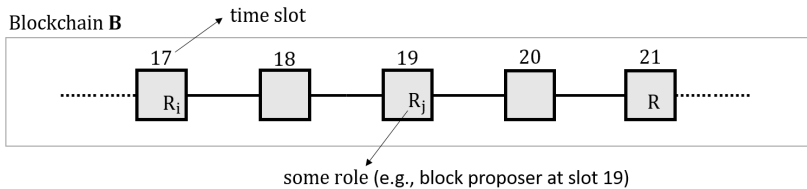
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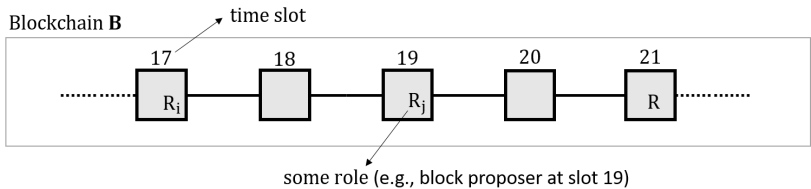
Contributions

Encryption to the Future [CDK+22]

Blockchain Lotteries

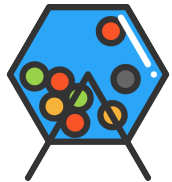


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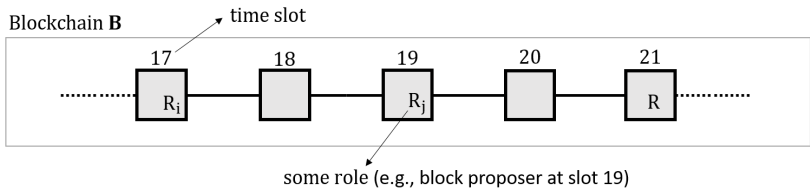


A self-selection mechanism that gives the winner the right to play a role R:

- propose a new block for the chain

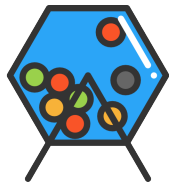


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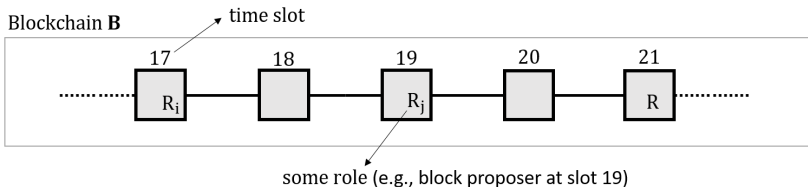


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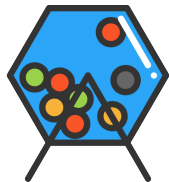


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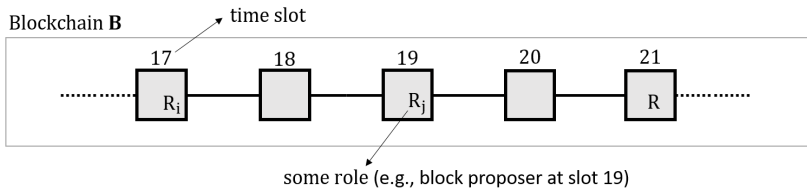


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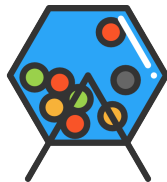
- propose a new block for the chain
- introduce new randomness
- become a member of a committee



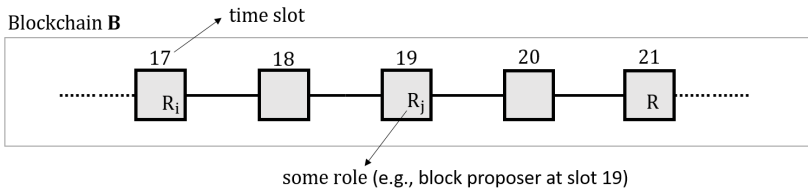
Blockchain Lotteries



Lottery Predicate. $\text{lottery}(\mathbf{B}, \text{slot}, R, sk_i) \in \{0, 1\}$

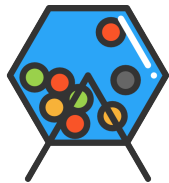


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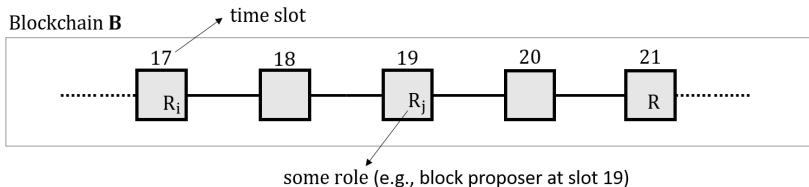


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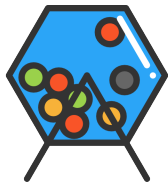


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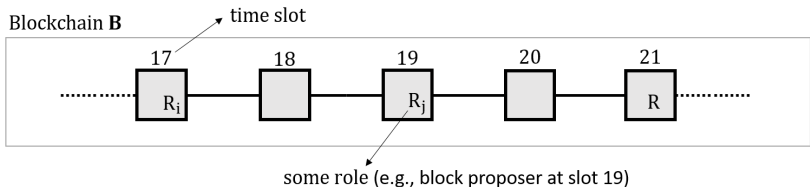


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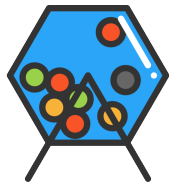


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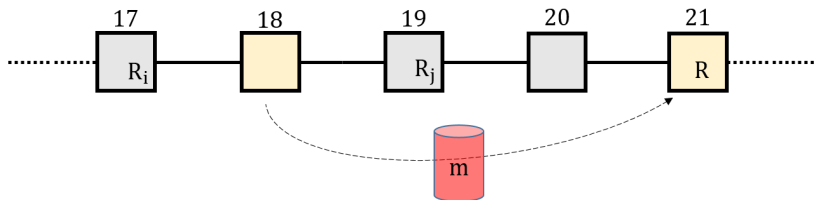


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 - No adaptive security!



Encryption to the Future

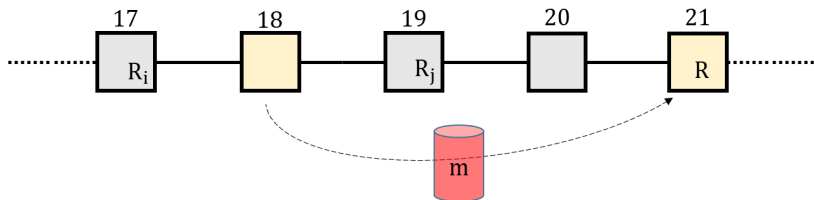


Encryption to the Future (EtF) w.r.t. $\text{lottery}(\mathbf{B}, \text{slot}, R, \text{sk})$.

Encryption. $\text{ct} \leftarrow \text{Enc}(\hat{\mathbf{B}}, \text{slot}, R, m)$

Decryption. $m/\perp \leftarrow \text{Dec}(\tilde{\mathbf{B}}, \text{ct}, \text{sk})$
Outputs m iff $\text{lottery}(\tilde{\mathbf{B}}, \text{slot}, R, \text{sk}) = 1$

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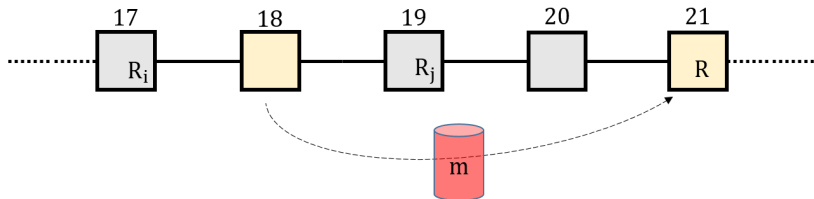
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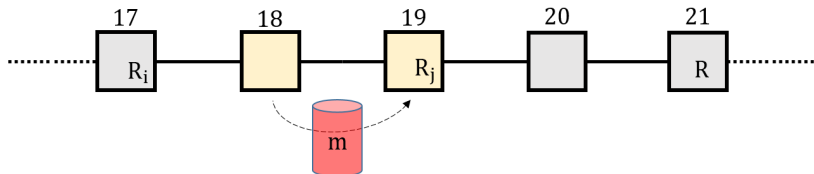
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$\hat{\mathbf{B}} \neq \tilde{\mathbf{B}}$ such that $\hat{\mathbf{B}}^{\uparrow \kappa} \preceq \tilde{\mathbf{B}}$ (far future)

stake distribution is unknown at encryption time.

"Harder" to realize, similar to [GKM⁺20] and implies Blockchain WE.

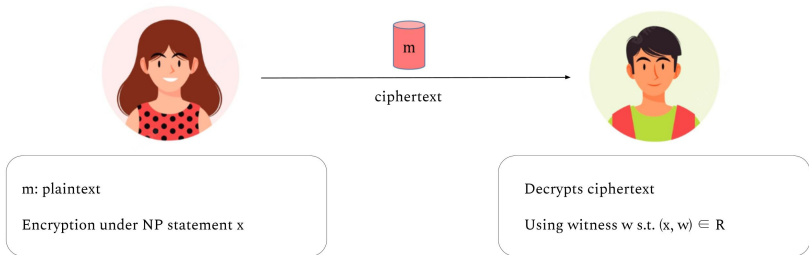
Encryption to the Future



Weaker Notion: Encryption to the Near Future

- Encryption w.r.t. lottery($\tilde{\mathbf{B}}$, slot, R_j , sk)
- The state of blockchain when the lottery winner is decided is known at the time of encryption: $\hat{\mathbf{B}} = \tilde{\mathbf{B}}$
- Can be constructed from "Witness Encryption over Commitments" (cWE)

Witness Encryption [?]



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A Witness Encryption scheme for **NP** language \mathcal{L} (and witness relation $\mathbf{R}_{\mathcal{L}}$).

Encrypt. $ct \leftarrow \text{Enc}(x, m)$,

Decrypt. $m/\perp \leftarrow \text{Dec}(ct, w)$

Properties:

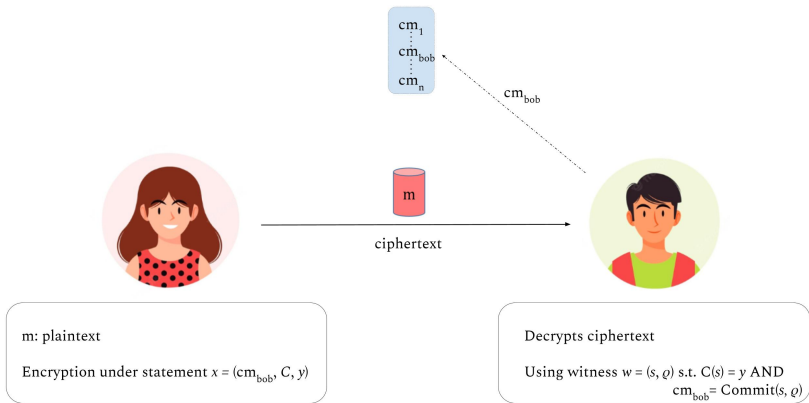
- Correctness: For any $x \in \mathcal{L}$ such that $(x, w) \in \mathbf{R}_{\mathcal{L}}$

$$\Pr [\text{Dec}(\text{Enc}(x, m), w) = m] = 1$$

- Security: For any PPT A , if $x \notin \mathcal{L}$ then

$$\Pr [A(\text{Enc}(x, 0)) = 1] - \Pr [A(\text{Enc}(x, 1)) = 1] \leq \text{negl}(\lambda)$$

Witness Encryption over Commitments (cWE)



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Setup Phase. Bob publishes a re-usable commitment
 $cm_{bob} \leftarrow \text{Commit}(ck, s; \rho)$

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Setup Phase. Bob publishes a re-usable commitment

$$cm_{bob} \leftarrow \text{Commit}(ck, s; \rho)$$

Encrypt Phase. Define a language of statements $x = (\text{com}, C, y)$ and witnesses $w = (s, \rho)$.

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3. "The adversary has no advantage in guessing m if it cannot point to a commitment of a satisfying s where it knows the opening."

Encryption to the (near) Future

Obtain EtF (near) from Witness Encryption over Commitments (cWE)

Setup. Let each party publish a commitment $cm_i \leftarrow \text{Commit}(sk_i; \rho)$ of the their lottery key

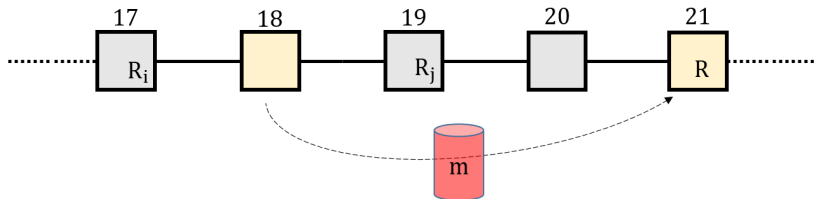
Encrypt. Let the circuit C encode the predicate $\text{lottery}(\mathbf{B}, \text{slot}, R, \cdot)$.
Use the statement $x_i = (cm_i, C, 1)$ for encryption.

Decrypt. The lottery-winning party with sk_i successfully decrypts since $C(sk_i) = 1$.

Result:

- The first non-interactive (using no auxiliary committees) Role Assignment protocol.
- Encryption has to be done "towards" every potential lottery winner—ciphertext length $O(N)$.
- For additional candidate constructions - read the paper.

Encryption to the Future



Encryption to the Future (EtF) w.r.t. $\text{lottery}(\mathbf{B}, \text{slot}, R, \text{sk})$.

Encryption. $\text{ct} \leftarrow \text{Enc}(\hat{\mathbf{B}}, \text{slot}, R, m)$

Decryption. $m/\perp \leftarrow \text{Dec}(\tilde{\mathbf{B}}, \text{ct}, \text{sk})$

Outputs m iff $\text{lottery}(\tilde{\mathbf{B}}, \text{slot}, R, \text{sk}) = 1$

$\hat{\mathbf{B}} = \tilde{\mathbf{B}}$ (near future)

blockchain state is unchanged (known stake distribution)

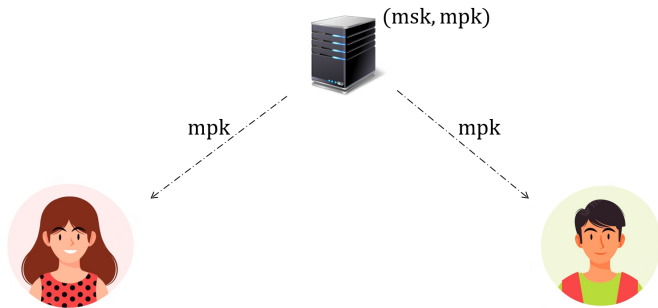
$\hat{\mathbf{B}} \neq \tilde{\mathbf{B}}$ such that $\hat{\mathbf{B}}^{\uparrow \kappa} \preceq \tilde{\mathbf{B}}$ (far future)

stake distribution is unknown at encryption time.

"Harder" to realize, similar to [GKM+20] and implies Blockchain WE.

Easy to realize using EtF (near future) + TIBE scheme and use of auxiliary committees

Identity Based Encryption (IBE)



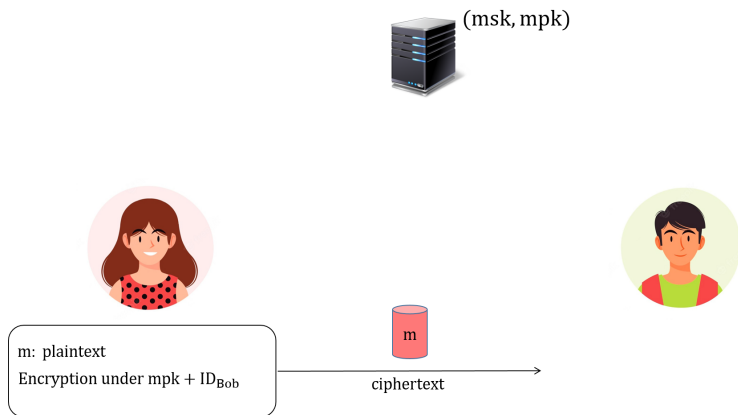
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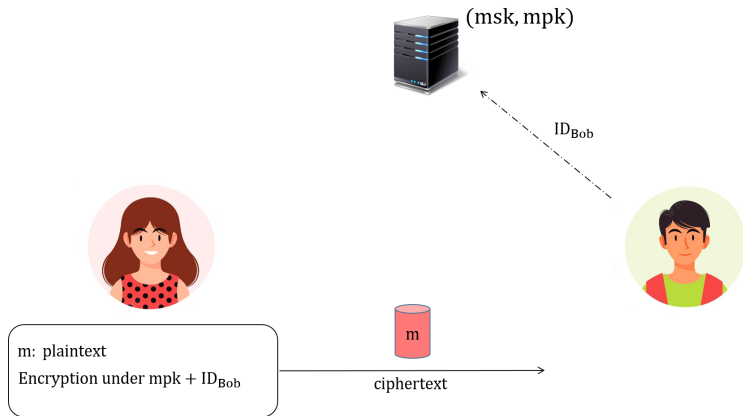
m: plaintext
Encryption under $mpk + ID_{Bob}$



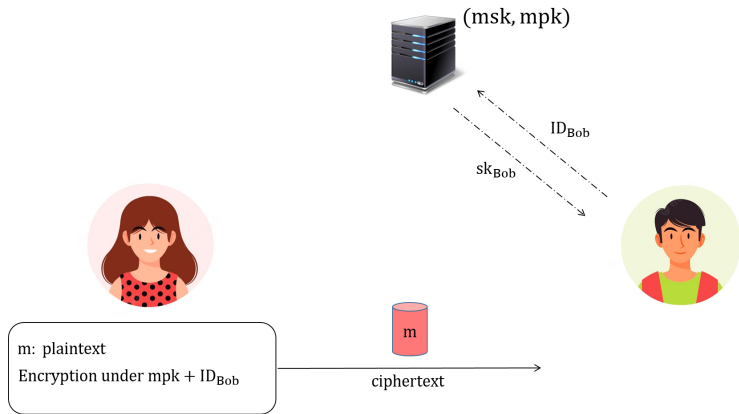
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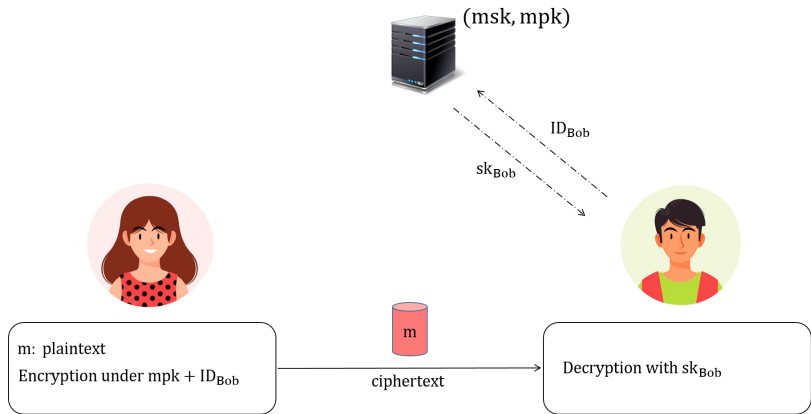
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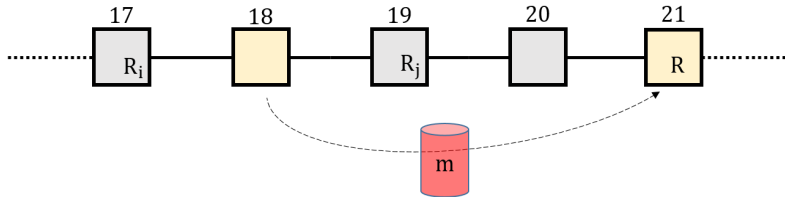
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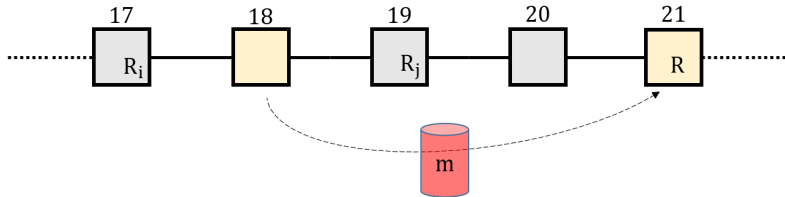
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Encryption to the Future with Committee

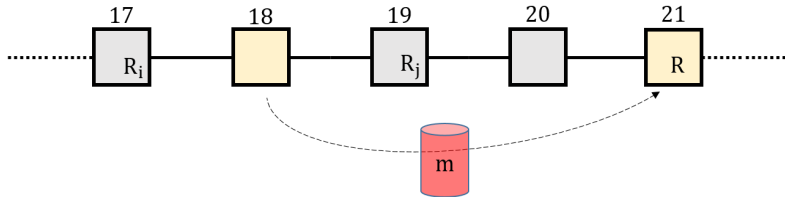


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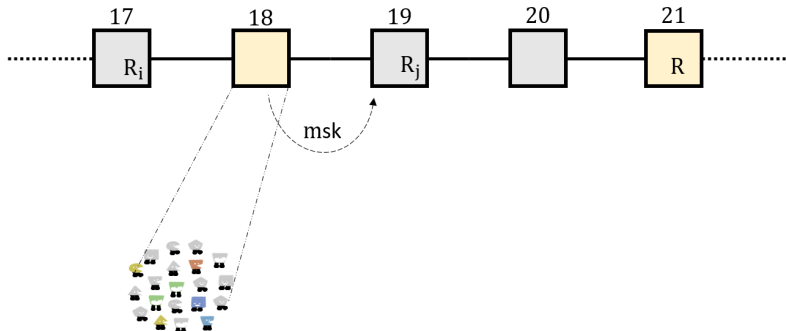
- **Encrypt.** Party publishes $ct \leftarrow \Pi_{\text{TIBE}}.\text{Enc}(\text{mpk}, \text{ID} = (\text{slot}, R), m)$.

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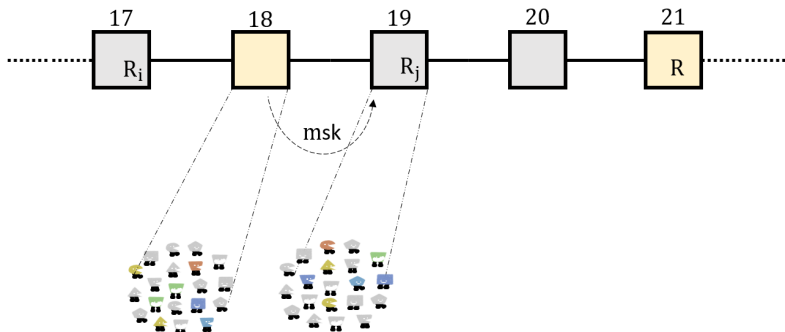
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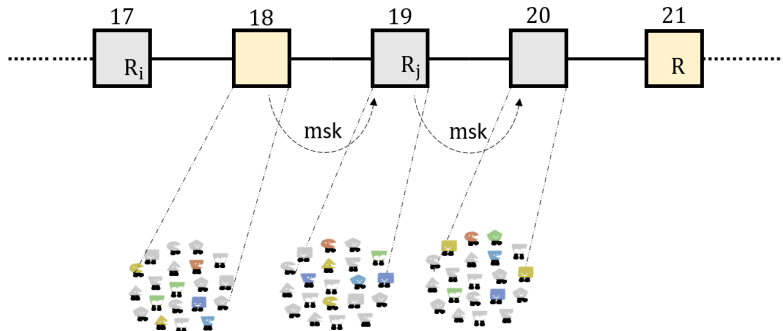
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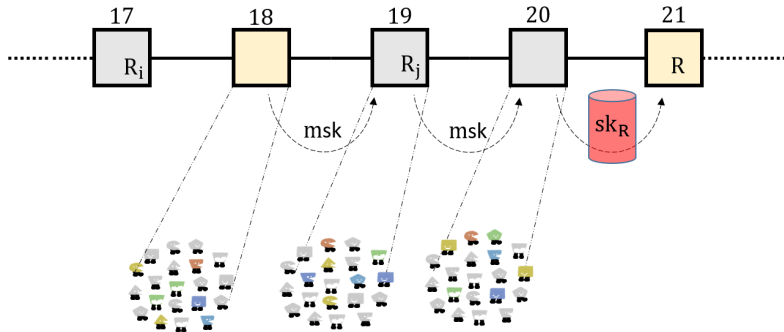
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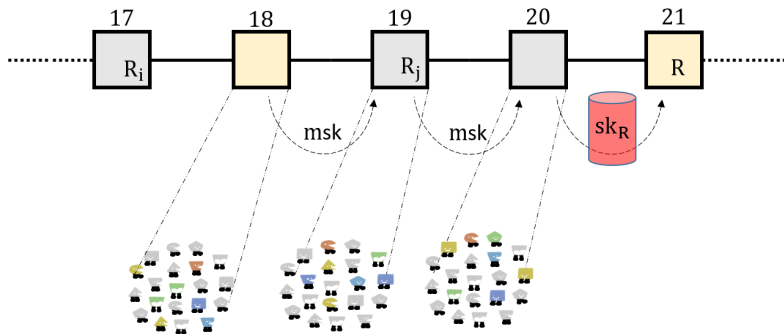
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 $sk_{(slot,R)}^i \leftarrow \Pi_{TIBE}.IDKeygen(msk_i, (slot, R))$
 - Send shares of ID-key by EtF (near) $ct_{(slot,R)}^{sk,i} \leftarrow \Pi_{EtF}.Enc(B, slot, R, sk_{(slot,R)}^i)$

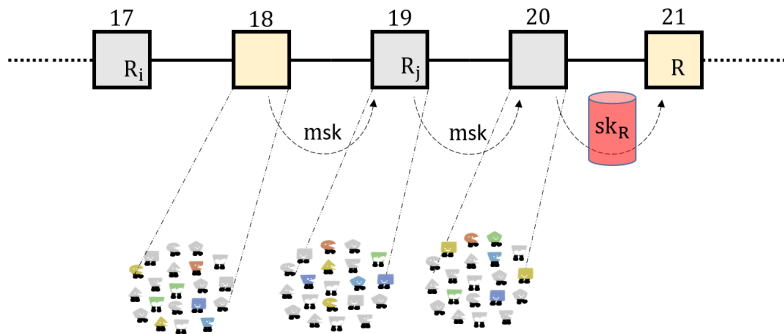
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Decrypt. The lottery-winner for (slot, R) decrypts EtF (near) ciphertexts and combine shares $\{sk_{(\text{slot}, R)}^i\}$ to obtain $sk_{(\text{slot}, R)}$. Finally outputs $m \leftarrow \Pi_{\text{TIBE}}.\text{Dec}(sk_{(\text{slot}, R)}, ct)$.

YOLO YOSO [CDGK22]

YOLO YOSO: EtF (near) using Shuffling

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- This is good enough for EtF, but how about secret sharing to a committee? How do we prove consistency between shares?

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- And simple proof of correct \mathbb{Z}_p -Linear decryption (e.g. ElGamal)

- **DH-PVSS:**

- We present the first PVSS with **constant size overhead**.
- The dealer has an initial key-pair (pk_D, sk_D) to enable the "SCRAPE check".
- Secret $S = s \cdot G$ is in group $\mathbb{G} = \langle G \rangle$ of order p , where DDH is hard.

We present two constructions of PVSS:

- **HE-PVSS:**

- Generic PVSS from a \mathbb{Z}_p -Linearly Homomorphic Encryption (LHE) scheme.
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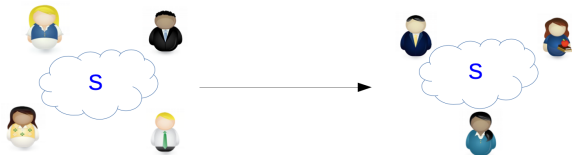
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Resharing and Reconstruction

Resharing:

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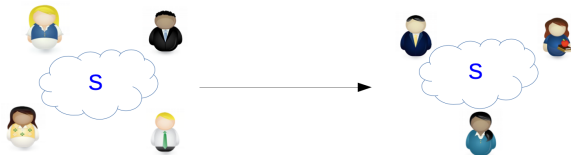


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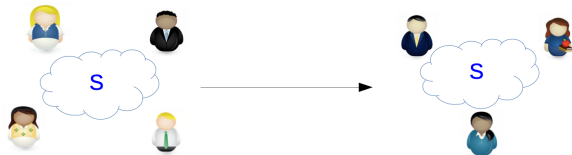
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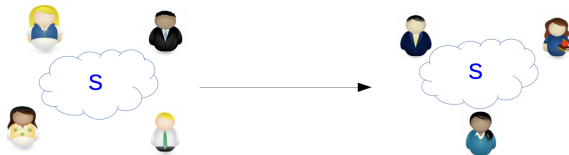
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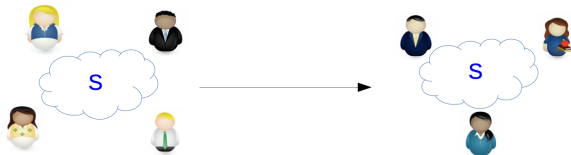
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4. $R_{r,i}$ that $C_{i \rightarrow j}$ are encryptions of a correct sharing whose secret is plaintext of $C_{r,i}$.

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Resharing and Reconstruction

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- Assuming proof of correct resharing, this implies proofs of correct reconstruction if $(n_{\text{last}} = 1, t_{\text{last}} = 0)$.
- Applications include:
 - "Keeping secrets on a blockchain" [BGG⁺20].
 - EtF (far) future (carrying the TIBE key).
 - Distributed Randomness Generation (Beacons).



HE-PVSS:

- Generic PVSS from a \mathbb{Z}_p -Linearly Homomorphic Encryption (LHE) scheme.
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- We present a DL-based PVSS. First one (as far as we know) with **constant size overhead**.
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- **New:** Dealer will also have a key pair $(sk_D, PK_D = sk_D \cdot G)$.
- Shamir shares of $S = s \cdot G$ (dealer does not need to know s)

$$(A_i)_{i \in [n]} \leftarrow \text{GShamir.Share}(pp_{\text{Sh}}, S)$$

- A_i encrypted as $C_i = A_i + sk_D \cdot PK_i$ ($sk_D \cdot PK_i$ is shared DH key).

Algorithm 8 $\text{GShamir.Share}(pp, S)$

- 1: **Input:** $S \in \mathbb{G}$
 - 2: $m(X) \xleftarrow{\$} \{m(X) \in \mathbb{Z}_p[X]_{\leq t} : m(\alpha_0) = 0\}$
 - 3: $A_i = S + m(\alpha_i) \cdot G, i \in [n]$
 - 4: **return** (A_1, \dots, A_n)
-

"SCRAPE Test" (Cascudo, David - ACNS17 [CD17]):

Theorem (SCRAPE dual-code test)

Let $1 \leq t < n$ be integers. Let p be a prime number with $p \geq n$. Let $\alpha_1, \dots, \alpha_n$ be pairwise different points in \mathbb{Z}_p . Define the coefficients $v_i = \prod_{j \in [n] \setminus \{i\}} (\alpha_i - \alpha_j)^{-1}$. Let

$$C = \{(m(\alpha_1), \dots, m(\alpha_n)) : m(X) \in \mathbb{Z}_p[X]_{\leq t}\}.$$

Then, for every vector $(\sigma_1, \dots, \sigma_n)$ in \mathbb{Z}_p^n :

$$(\sigma_1, \dots, \sigma_n) \in C \Leftrightarrow \sum_{i=1}^n v_i \cdot m^*(\alpha_i) \cdot \sigma_i = 0, \quad \forall m^* \in \mathbb{Z}_p[X]_{\leq n-t-1}.$$

"SCRAPE Test" (Cascardo, David - ACNS17 [CD17]):

- $\mathcal{C} = \{(m(1), \dots, m(n)) : \deg(m) \leq t\}$ is a linear (Reed-Solomon) code space.

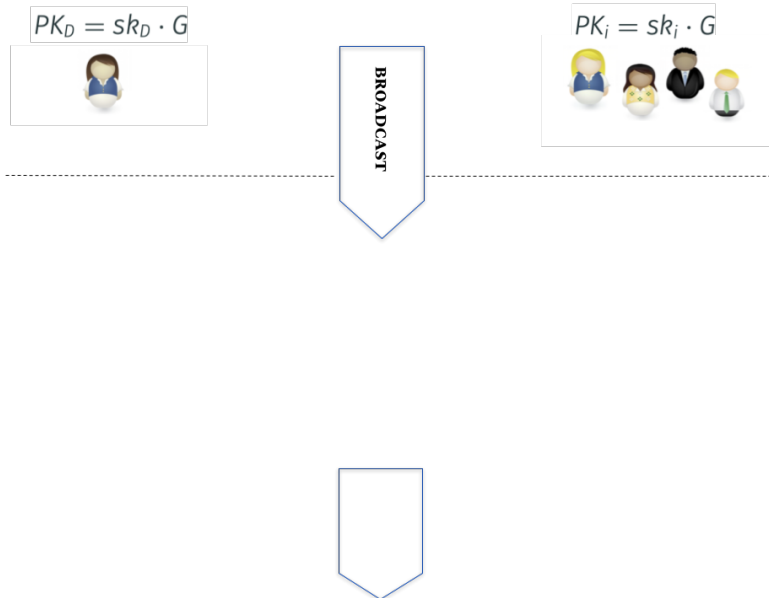
"SCRAPE Test" (Cascardo, David - ACNS17 [CD17]):

- $\mathcal{C} = \{(m(1), \dots, m(n)) : \deg(m) \leq t\}$ is a linear (Reed-Solomon) code space.
- It has a dual code space:
 $\mathcal{D} = \{(m^*(1), \dots, m^*(n)) : \deg(m^*) \leq n - t - 1\}$.
- Let $\mathbf{a} = (a_1, \dots, a_n)$ in $(\mathbb{Z}_p)^n$. Sample $\mathbf{d} = (d_1, \dots, d_n)$ from \mathcal{D}
 - If $\mathbf{a} \in \mathcal{C}$, then $\sum_{i=1}^n v_i \cdot d_i \cdot a_i = 0$.
 - If $\mathbf{a} \notin \mathcal{C}$, then $\sum_{i=1}^n v_i \cdot d_i \cdot a_i = 0$, with probability $1/p$.

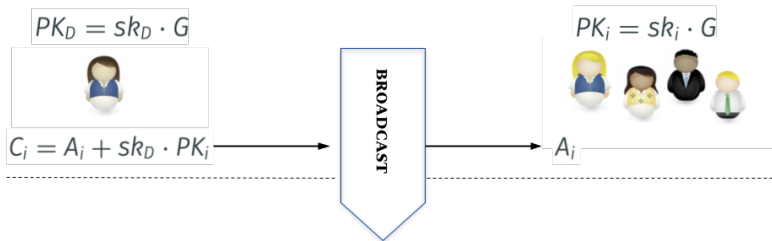
Extends to group $\mathbb{G} = \langle G \rangle$ where $A_i = a_i \cdot G$:

- Given (A_1, \dots, A_n) in \mathbb{G}^n . Sample $\mathbf{d} = (d_1, \dots, d_n)$ from \mathcal{D} .

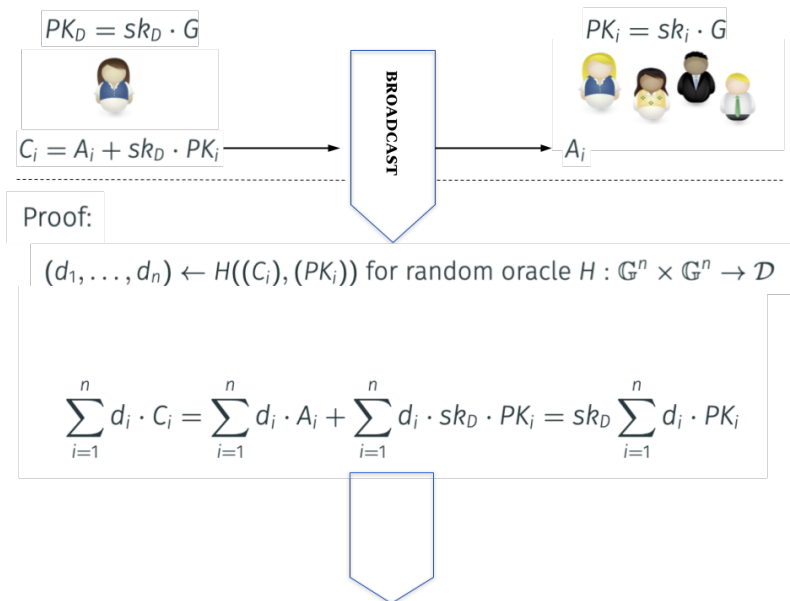
$$\sum_{i=1}^n v_i \cdot d_i \cdot A_i \stackrel{?}{=} O,$$



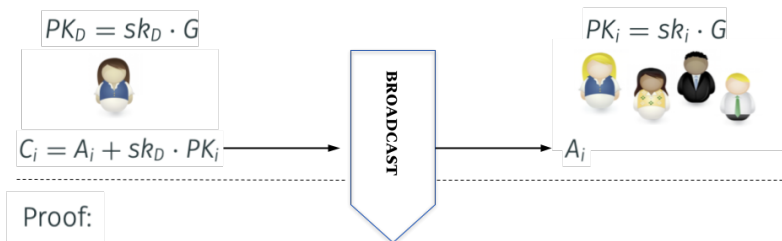
DH-PVSS - Ideas 2



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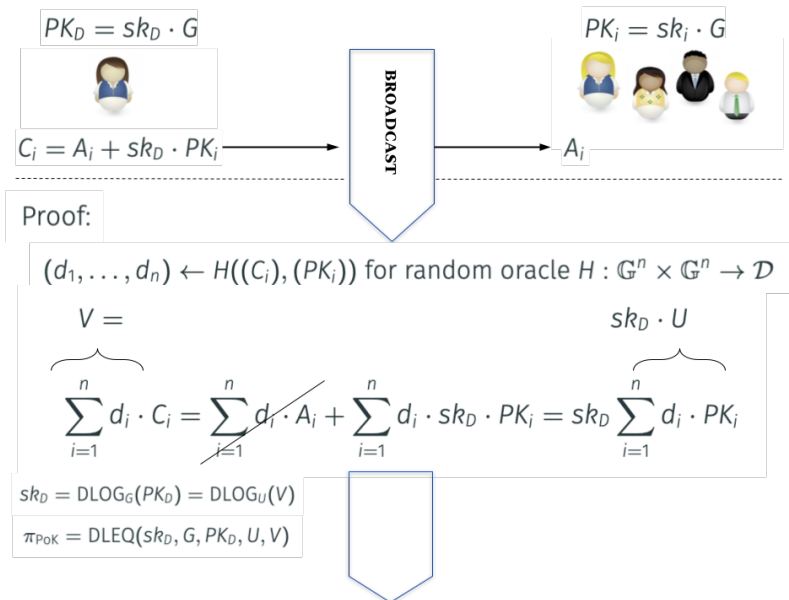


Proof:

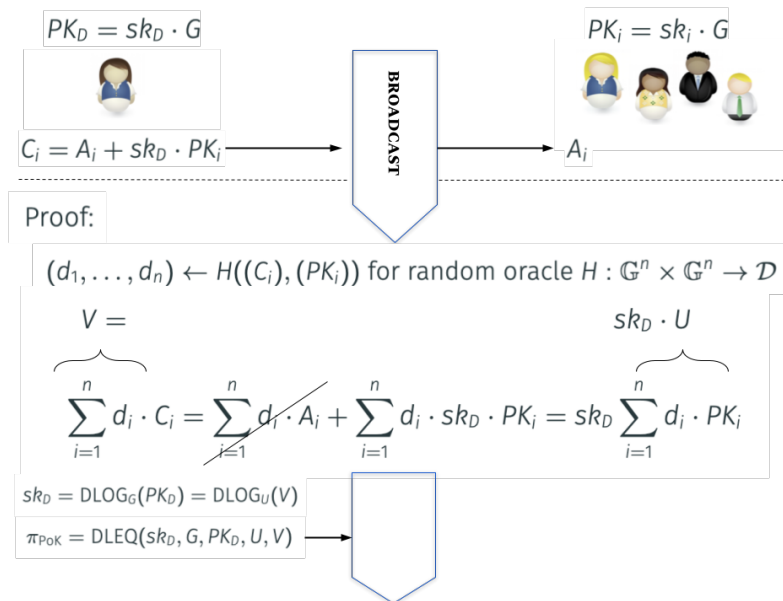
$(d_1, \dots, d_n) \leftarrow H((C_i), (PK_i))$ for random oracle $H : \mathbb{G}^n \times \mathbb{G}^n \rightarrow \mathcal{D}$

$$V = \sum_{i=1}^n d_i \cdot C_i = \sum_{i=1}^n d_i \cdot A_i + \sum_{i=1}^n d_i \cdot sk_D \cdot PK_i = sk_D \sum_{i=1}^n d_i \cdot PK_i = sk_D \cdot U$$

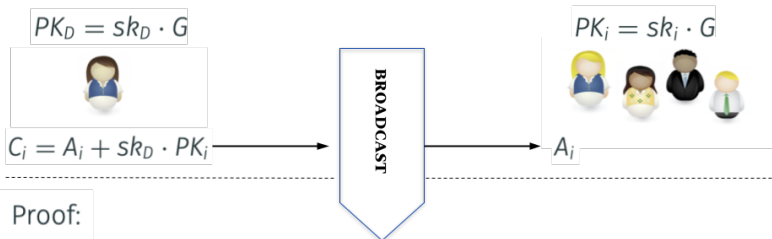
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$$sk_D = \text{DLOG}_G(PK_D) = \text{DLOG}_U(V)$$

$$\pi_{\text{PoK}} = \text{DLEQ}(sk_D, G, PK_D, U, V)$$

$\text{Verify}(pp, pk_D, \{(pk_i, C_i) : i \in [n]\}, \pi_{\text{PoK}})$

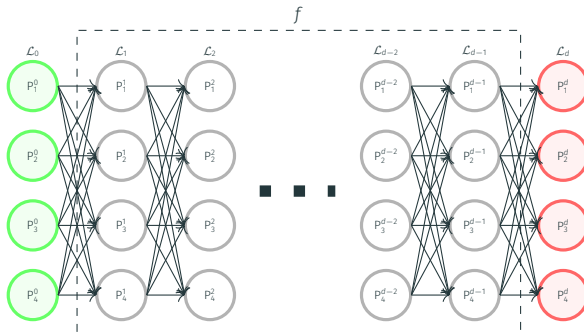
Layered MPC [DDG⁺23]

Layered MPC

An (n, t, d) -layered protocol has the following properties:

Parties. $N = n(d + 1)$ parties partitioned into $d + 1$ layers \mathcal{L}_i , $0 \leq i \leq d$, where $|\mathcal{L}_i| = n$.

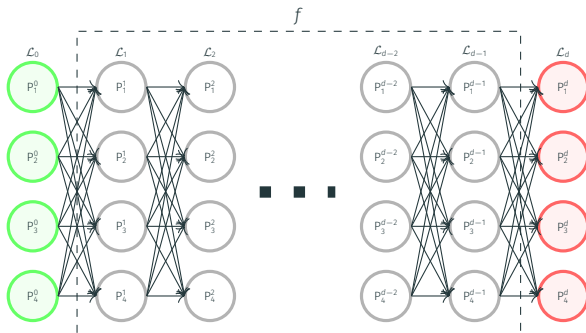
Interaction. d synchronous rounds where parties in \mathcal{L}_{i-1} may send messages to parties in \mathcal{L}_i over secure channels and broadcast.



Layered MPC

Functionalities. We consider functionalities f that take inputs from **input clients** and deliver outputs to **output clients**.

Adversaries. We consider active, rushing, adaptive adversaries who may corrupt any number of input/output clients, and t parties in layers \mathcal{L}_i , $0 < i < d$.



A note on Layered Broadcast

- The model of layered MPC assumes layer-to-layer broadcast.
- Deterministic Broadcast is impossible in the layered setting.
- Derived from the result of [Gar94] on reaching agreement in the mobile setting.

Lemma 2

Deterministic Broadcast is possible iff $t = 0$.

Basic Primitives

Future Messaging

Future Messaging functionality f_{FM}

PUBLIC PARAMETERS: Sender $S \in \mathcal{L}_0$, receiver $R \in \mathcal{L}_d$ for $d > 0$ and message domain M .

SECRET INPUTS: S has input $m \in M$.

f_{FM} receives m from S , and delivers m to R .

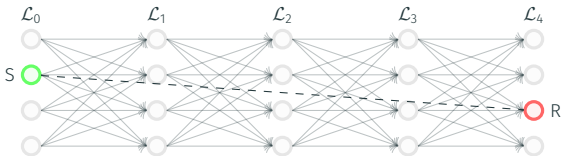
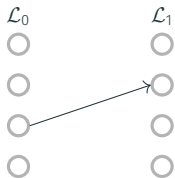


Figure 1: Π_{FM} from S of m to R

Future Messaging

Π_{FM} from \mathcal{L}_0 to \mathcal{L}_1 :

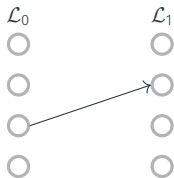
Use the secure point-to-point channels from layer to the next layer.



Future Messaging

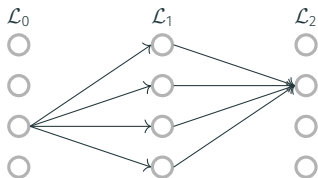
Π_{FM} from \mathcal{L}_0 to \mathcal{L}_1 :

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Π_{FM} from \mathcal{L}_0 to \mathcal{L}_2 :

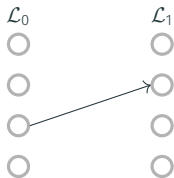
1. S does $\text{Sh}(m) = (s_1, \dots, s_n)$ and sends s_j to P_j^1 .
2. P_j^1 forwards s_j to R and R obtains $\hat{m} = \text{Rec}(\hat{s}_1, \dots, \hat{s}_n)$



Future Messaging

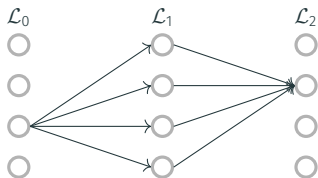
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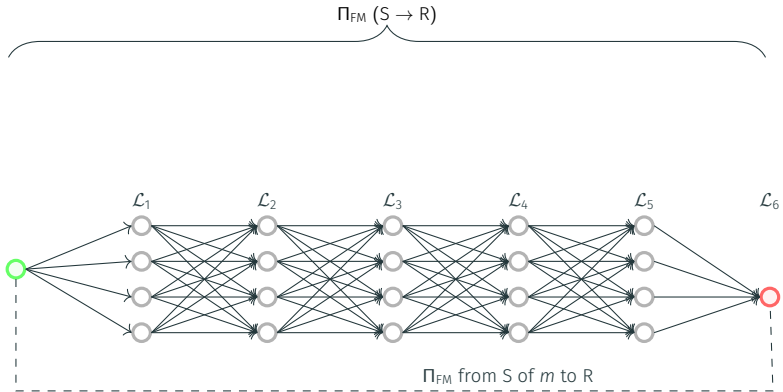
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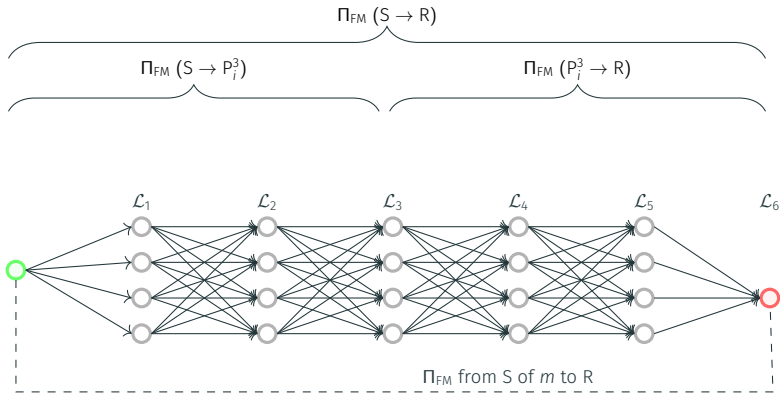


(Equivalent to perfect malicious 1-way SMT [DDWY93])

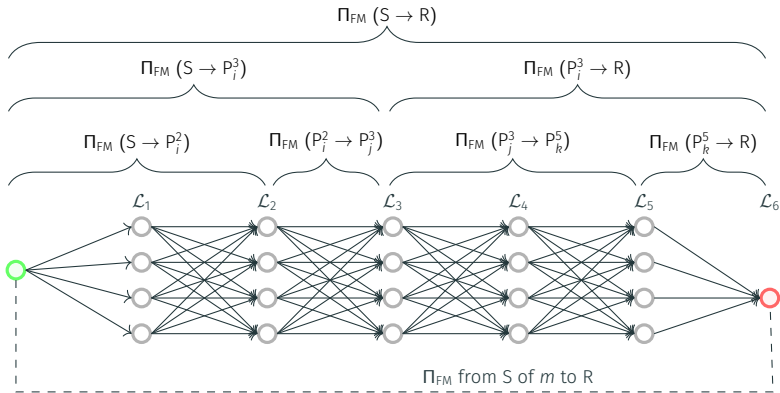
Future Messaging



Future Messaging



Future Messaging

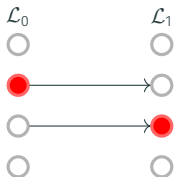


Future Messaging (and Rushing)

Dishonest Sender and problems with rushing

PARALLEL INVOCATIONS f_{FM}^n :

- When invoking multiple f_{FM} in parallel, the adversary can cause a **correlation attack**.
- Model the parallel functionality as **corruption-aware**.

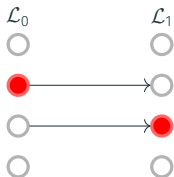


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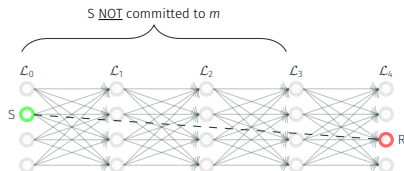
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NON-COMMITTING PRIMITIVE:

- The adversary can change the message m to a message of its choosing m' in f_{FM} until the last round.
- Where YOSO assumes ideal **committing** communication to future rounds.



Future Broadcast

(Conditional) Future Broadcast

- FUTURE BROADCAST:
Invoke f_{FM} where parties in \mathcal{L}_{d-1} are instructed to broadcast their shares instead of sending to a recipient R.
- CONDITIONAL DISCLOSURE:
Conditioned on some event E , honest parties in \mathcal{L}_{d-1} reveal their shares.

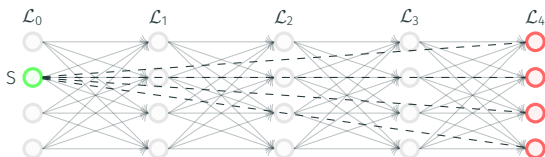


Figure 2: Future Broadcast from S of m to \mathcal{L}_4

Summary of Future Messaging:

Complexity Assuming a linear secret sharing scheme, Π_{FM} is a recursive protocol realizing f_{FM} with communication complexity $O(n^{\lceil \log d \rceil} \log |M|)$.

Security Honest sender reduces to an instance of SMT Dishonest sender is challenging with rushing. Especially, when composing protocols.

Extension Future Messaging can be extended to (Conditional) Future Broadcast.

Towards Layered MPC

Layered CNF-VSS Protocol

CNF-VSS of [GIKR01]



Weak Future Multicast $\Pi_{\text{weak-FMcast}}$



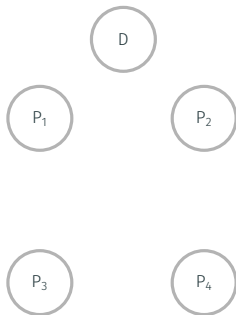
Future Multicast Π_{FMcast}



Verifiable Secret Sharing Π_{VSS}

4-round perfect CNF VSS

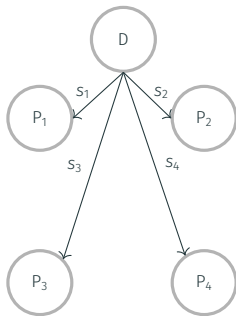
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4-round perfect CNF VSS

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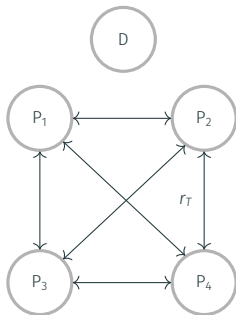
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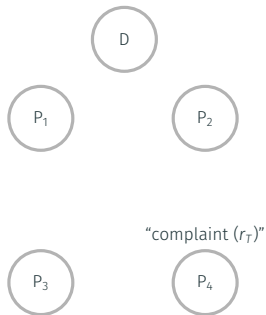
1. D sends $s_j = (r_T)_{T \ni j}$ to P_j .
2. Each pair $(P_j, P_{j'})$ exchange share r_T (if $j, j' \in T$).



4-round perfect CNF VSS

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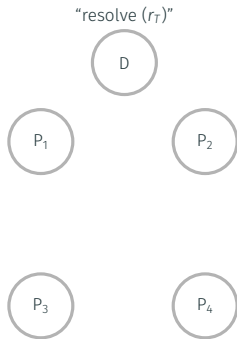
1. D sends $s_j = (r_T)_{T \ni j}$ to P_j .
2. Each pair $(P_j, P_{j'})$ exchange share r_T (if $j, j' \in T$).
3. If disagreement, involved parties broadcast “complaint (r_T) ”.



4-round perfect CNF VSS

D (dealer) holds a secret $s \in \mathbb{F}$ and obtains $\text{Sh}_{\text{CNF}}(s) = (s_1, \dots, s_n)$.

1. D sends $s_j = (r_T)_{T \ni j}$ to P_j .
2. Each pair $(P_j, P_{j'})$ exchange share r_T (if $j, j' \in T$).
3. If disagreement, involved parties broadcast “complaint (r_T) ”.
4. D then broadcasts “resolve (r_T) ”, if any complaints received from P_j or $P_{j'}$.



Challenges with layered [GIKR01]

- Dealer speaks more than once (round 1 and round 4).

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Solution:

Emulate the dealer using Conditional Future Broadcast.

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Emulate the dealer using Conditional Future Broadcast.

- P_j and $P_{j'}$ exchange additive shares.

Solution:

Invoke a Distributed Equality Check with Π_{add} for each pair (j, j') .

Future Multicast

Future Multicast functionality f_{FMcast}

PUBLIC PARAMETERS: Sender $S \in \mathcal{L}_0$, receiving set of parties

$R \subseteq \mathcal{L}_d, d \geq 5$, message domain M .

SECRET INPUTS: S has input $m \in M$.

f_{FMcast} receives m from S , and delivers m to all parties in R .

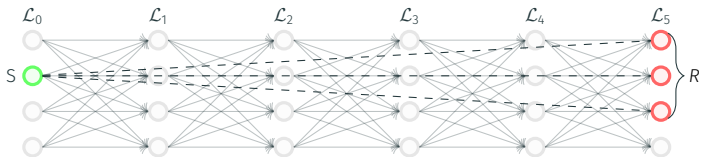
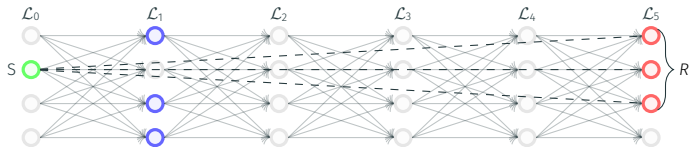


Figure 3: Π_{FMcast} from $S \in \mathcal{L}_0$ of m to $R \subseteq \mathcal{L}_5$

Future Multicast

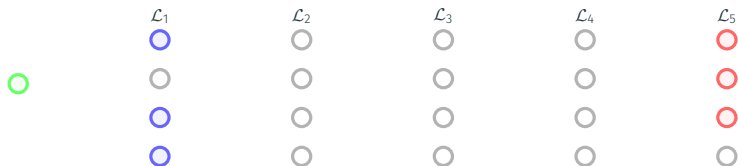
Sketch of Π_{FMcast}

1. S samples additive shares $\{r_T\}_{T \in \mathcal{T}}$ of m .
2. S sends each r_T to $R \subseteq \mathcal{L}_5$ using $\Pi_{\text{weak-FMcast}}$.
Using a different set of intermediaries $U_T \subset \mathcal{L}_1$ where $|U_T| = n - t$.
3. Parties in $R \subseteq \mathcal{L}_5$ do $\hat{m} = \sum_{T \in \mathcal{T}} \hat{r}_T$.



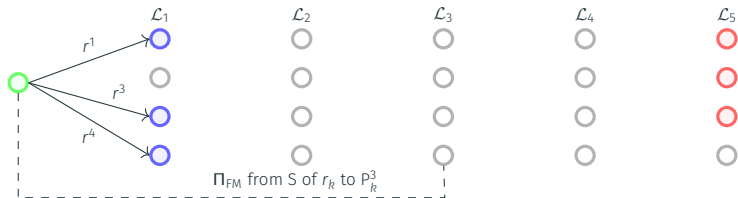
Weak Future Multicast

$\Pi_{\text{weak-FMcast}}$ of $r = r_T$ from $S \in \mathcal{L}_0$ to R using U_T as intermediaries.



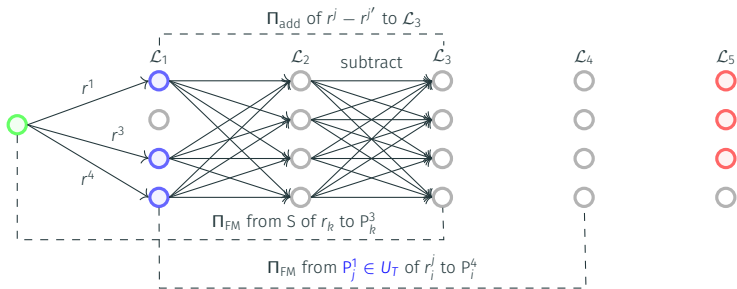
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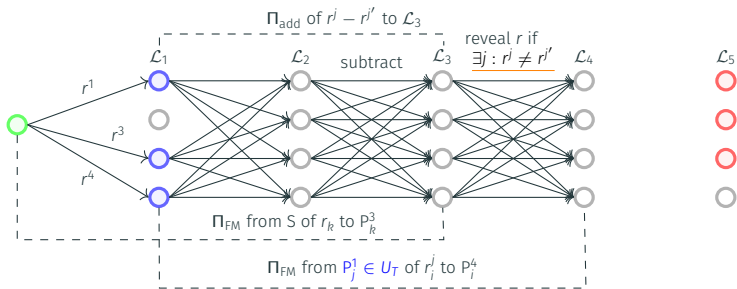
Weak Future Multicast

$\Pi_{\text{weak-FMcast}}$ of $r = r_T$ from $S \in \mathcal{L}_0$ to R using U_T as intermediaries.



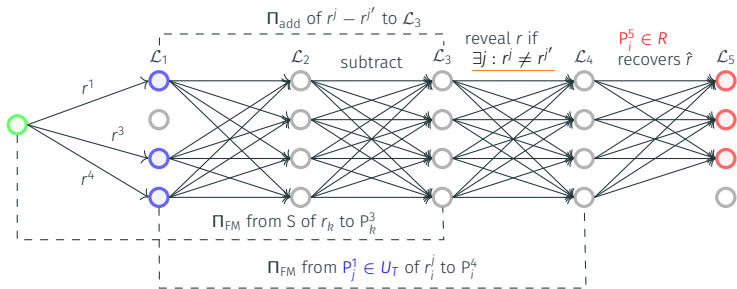
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Layered VSS

An $(n, t, 5)$ -layered protocol Π_{VSS} realizing f_{VSS} where $t < n/3$.

From $\Pi_{\text{weak-FMcast}}$ to Π_{FMcast} :

From Π_{FMcast} to Π_{VSS} :

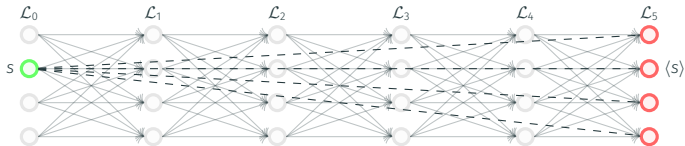


Figure 4: Π_{VSS} from $S \in \mathcal{L}_0$ of m to \mathcal{L}_5

Layered VSS

An $(n, t, 5)$ -layered protocol Π_{VSS} realizing f_{VSS} where $t < n/3$.

From $\Pi_{\text{weak-FMcast}}$ to Π_{FMcast} :

- Each additive share r_T is transferred to R using U_T .

From Π_{FMcast} to Π_{VSS} :

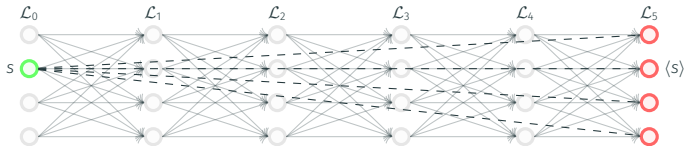


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From $\Pi_{\text{weak-FMcast}}$ to Π_{FMcast} :

- Each additive share r_T is transferred to R using U_T .
- Since at least one set (U_T) is comprised of only honest parties the message $m = \sum_{T \in \mathcal{T}} r_T$ remains secure if S and R are honest.

From Π_{FMcast} to Π_{VSS} :

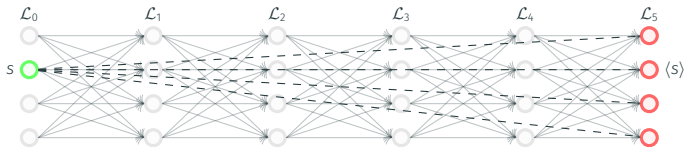


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From Π_{FMcast} to Π_{VSS} :

- S samples $\{r_T\}_{T \in \mathcal{T}}$ as additive secret sharing of secret s .

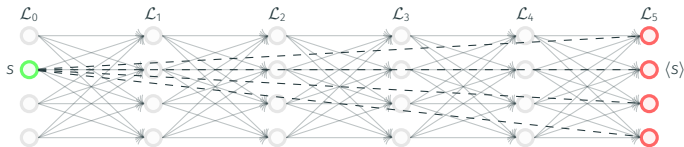


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From Π_{FMcast} to Π_{VSS} :

- S samples $\{r_T\}_{T \in \mathcal{T}}$ as additive secret sharing of secret s .
- For each $T \in \mathcal{T}$, execute Π_{FMcast} with S as sender with input r_T and $\{P_i^5 : i \in T\}$ as receivers.

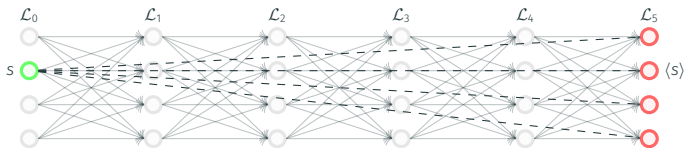


Figure 4: Π_{VSS} from $S \in \mathcal{L}_0$ of m to \mathcal{L}_5

Results

Theorem 1: CNF-Based Layered MPC

Let f be an n -party functionality computed by a **layered arithmetic circuit** C over a finite ring, with D **layers** and M gates. Then, for any $t < n/3$, there is an $(n, t, O(D))$ -**layered MPC protocol** for f . The communication consists of $2^{O(n)} \cdot M$ ring elements.

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Corollary 1: Secure Maximally Proactive MPC

Let f be an n -party functionality computed by a layered arithmetic circuit C over a finite ring, **with D layers**. Then, for $t < n/3$, there is a **maximally proactive MPC protocol** computing f in $r = O(D)$ rounds.

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Corollary 1: Secure Maximally Proactive MPC

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- May be concretely efficient for small n .
- Use techniques from [CDI05] to amortize the communication overhead by sending k -bit seeds and let the receivers generate most shares locally.
- This technique makes use of black-box access to PRG (computational security).

Theorem 2: Efficient Layered MPC

Let f be an n -party functionality computed by a **layered arithmetic circuit** C over a finite field, with D **layers** and M gates. Then, for any $t < n/3$, there is an $(n, t, O(D))$ -**layered MPC protocol** for f . The communication consists of $O(n^9) \cdot M$ field elements.

Theorem 2: Efficient Layered MPC

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Corollary 2: (Efficient) Secure Maximally Proactive MPC

Let f be an n -party functionality computed by a layered arithmetic circuit C over a finite field, **with D layers**. Then, for $t < n/3$, there is an efficient **maximally proactive MPC protocol** computing f in $r = O(D)$ **rounds**.

- Extending the techniques for Distributed Equality Check and Conditional Future Broadcast to the [BGW88]-setting.

Results

f	Reference	Level	Security	Comm.	Threshold
FM	This work	perfect	full	$\text{poly}(n)$	$t < n/3$
	[BGG ⁺ 20]	comp.	full	$\text{poly}(n)$	$t < n/4^*$
VSS	This work	perfect	full	$2^{O(n)}$	$t < n/3$
	This work (Sec. 5)	perfect	full	$\text{poly}(n)$	$t < n/3$
MPC	[GHK ⁺ 21] (YOSO)	statistical	full +setup [†]	$\text{poly}(n)$	$t < n/2^*$
	[CGG ⁺ 21] (Fluid)	statistical	abort	$\text{poly}(n)$	$t < n/2$
	[OY91]	perfect	full	$\text{poly}(n)$	$t < n/d$
	This work	perfect	full	$2^{O(n)}$	$t < n/3$
	This work (Sec. 5)	perfect	full	$\text{poly}(n)$	$t < n/3$
	This work (Sec. 6)	comp.	full	$\text{poly}(n)$	$t < n/2$

Table 1: Protocols realizing primitives in the most extreme proactive settings.
 (*protocol security relies on the adversary only doing probabilistic corruption,
[†]assumes access to ideal target-anonymous channels for future messaging)

Layer 1

PoS Blockchain (PKI + BC + lottery)

Ephemeral Committees Model



Ephemeral Committees Model

Layer 2 (Encryption to the Future):

- Formalize and define Encryption to the Future
- Construct EtF (near) using only OT+GC (w/o auxiliary committees)
- Construct EtF (far) using TIBE w/ comm. independent of M .

Conclusion

Layer 3 (YOLO-YOSO):

- Mixnet-based EtF (near) using standard PKE
- Propose a generic HE-PVSS and a extremely efficient DH-PVSS.
- Both HE-PVSS and DH-PVSS extendable to proactive resharing (YOSO).

Layer 2 (Encryption to the Future):

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Layer 3

Public VSS w/ Ephemeral Parties [CDGK22]

Layer 2

Communication [CDK⁺22]

Layer 1

PoS Blockchain (PKI + BC + lottery)

Ephemeral Committees Model

Conclusion

Layer 4 (Layered MPC):

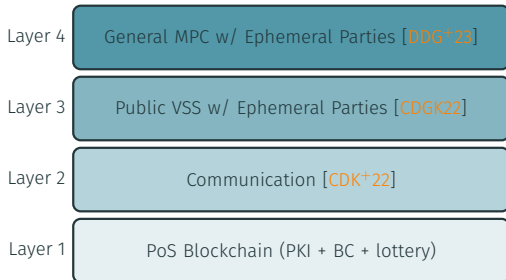
- MPC w/ restricted interaction.
- Prove feasibility of general perfect MPC with $n/3$.
- Show implications to classic proactive MPC and newer YOSO.

Layer 3 (YOLO-YOSO):

- Mixnet-based EtF (near) using standard PKE
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Ephemeral Committees Model

Thank You!

Link to Eprints:


<https://ia.cr/2021/1423>

<https://ia.cr/2022/242>

<https://ia.cr/2023/330>

Link to Thesis:

<https://akonring.github.io/thesis.pdf>

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